An Empirical Investigation of Changes in Scale Economies for the Commercial Banking Firm, 1979-1986

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THE 1980s REPRESENT A PERIOD of technological and regulatory change perhaps unparalleled in American banking history. There have been tremendous advances in technology from the computer and communications industries. Regulatory restrictions regarding the types of services that banks and other depository institutions may offer have changed, blurring the traditional distinctions between financial firms. The Regulation Q restrictions on explicit interest payments on some types of deposits were phased out. In addition, banks face growing competition from other firms seeking to provide bank-like services. Geographic restrictions, which previously limited both the number of competitors banks faced and the organizational structure which banks used, have changed markedly. These technological and regulatory changes are likely to affect the cost structure of commercial banks.

This study examines data from 1979 to 1986 in order to update previously published results, and to determine the direction and magnitude of the possible shifts in the structure of bank costs. In particular, the focus here is on economies of scale for smaller and medium-sized banks.

This paper is divided into the following sections. A brief summary of the current literature is given, followed by a description of the model used in this study, the data on which the estimations were conducted, and the analytical results.

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LITERATURE SUMMARY

While the topic of cost structure of firms in the financial services industry has generated a substantial literature, some recent studies report results that stand at odds with earlier findings. The conclusions of some of the relatively recent studies may be of somewhat limited usefulness for current public policy guidance because they analyze data from the 1970s,[1] although these are several studies which analyze data from the early 1980s.[2] There are certain aspects of cost structure where a consensus appears to have been reached, and others where considerable variation exists. State regulations on bank structure, in the form of branching restrictions, appear to matter. There still exists some question in the literature as to whether scale economies exist beyond some small level of output. While most of these studies conclude that any scale economies are exhausted at relatively low levels of output, Hunter and Timme (1986) and Lawrence and Shay (1986) found economies of scale across a broad range of bank sizes. In a recent study by Noulas, Ray, and Miller (1990) economies of scale were found for banks up to $3 billion in total assets. These contrasting results may be due to the different data sets used in the empirical estimations, alternative model specifications, and some model misspecifications.[3]

In almost all of the multiproduct cost studies referred to above, some version of the transcendental logarithmic cost function was used to model bank costs. The translog cost model is presented briefly below.

THE MODEL

Production in the multiproduct firm can be represented by the transformation function:

$$f_1(Y_1, Y_2, \ldots Y_m; X_1, X_2, \ldots X_n) = 0$$

where Y_j are the m outputs and X_i are the n inputs. If this function is strictly convex in inputs X_i, duality insures that there exists a unique corresponding cost function:

$$C = f_2(Y_1, Y_2, \ldots Y_m; P_1, P_2, \ldots P_n)$$

where P_i are the prices paid for the X_i inputs, and Y_j are the m outputs. This cost function is homogeneous of degree one and concave in factor prices, and non-decreasing in both factor prices and output quantities. These restrictions insure that there exists a unique correspondence between the cost function and the underlying

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[1]These include Benston, Hanweck, and Humphrey (1982), Clark (1984), Gilligan and Smirlock (1984), Gilligan, Smirlock, and Marshall (1982, 1984), Murray and White (1983), and Nelson (1985).

[2]For a recent survey of this literature, see Clark (1988).

[3]See Zardkoohi, Rangan, and Kolari (1986) on these misspecifications.

production function.[4] The cost function describes the production process as completely as does the transformation function, and thus cost function estimates can be used to make inferences about production characteristics.

For empirical estimation, a flexible functional form that imposes minimal restrictions consistent with a dual relation between production and cost is desired. The translog cost function is essentially a second-order expansion in input prices and output quantities, and thus it provides a second-order approximation to an arbitrary cost function.[5] Translog cost curves are not restricted to the monotonically increasing or decreasing shapes imposed by the Cobb-Douglas or CES specifications.[6] This functional form has been applied to the study of financial firms by several researchers.[7]

The general form of the translog cost function is shown below:

$$\ln C = \alpha_0 + \Sigma_i \, \alpha_i \, \ln Y_i + \Sigma_j \, \beta_j \, \ln P_j + \tfrac{1}{2} \Sigma_i \, \Sigma_k \, \sigma_{ik} \, \ln Y_i \, \ln Y_k$$

$$+ \tfrac{1}{2} \Sigma_j \, \Sigma_h \, \delta_{jh} \, \ln P_j \, \ln P_h + \Sigma_i \, \Sigma_j \, \tau_{ij} \, \ln Y_i \, \ln P_j \, . \tag{1}$$

The Y_i represent the n outputs, and the P_j represent the m input prices. Linear homogeneity in input prices is insured by the restrictions $\Sigma_j \, \beta_j = 1$, $\Sigma_j \, \delta_{jh} = 0$, and $\Sigma_j \, \tau_{ij} = 0$. This cost function can be estimated alone, or the factor share equations can be derived using Shephard's lemma, and the system estimated simultaneously. The factor share equations are as shown below:

$$\partial \ln C / \partial \ln P_j = S_j = \beta_j + \Sigma_j \, \delta_{jh} \, \ln P_h + \Sigma_j \, \tau_{ij} \, \ln Y_i \tag{2}$$

where S_j is the share of expenditures on the jth input in total cost.

Because of the restriction of linear homogeneity in input prices, the factor share equations must sum to one. To avoid singularity problems, one of the share equations must be excluded in the estimation process. The parameter estimates are invariant with respect to which equation is excluded from the estimated system.[8] The overall scale economies (OSE) realized when all outputs are increased by a common factor can be obtained by differentiating the cost function with respect to all Y_i.

$$OSE = \Sigma_i \, \alpha_i + \Sigma_i \, \sigma_{ik} \, \ln Y_k + \Sigma_i \, \tau_{ij} \, \ln P_j \, . \tag{3}$$

[4]For an elaboration on this point, see Diewert (1973).

[5]For a discussion of the translog functional form see Christensen, Jorgensen, and Lau (1973), Denny and Pinto (1978), or Diewert (1973).

[6]The restrictions imposed by the Cobb-Douglas or CES forms can be tested by restricting certain parameters in the translog model. Thus constant elasticity of substitution can be a testable, rather than a maintained hypothesis. Several studies have conducted tests of this and other restrictions. See Clark (1984) and Lawrence (1989).

[7]These include Benston, Hanweck, and Humphrey (1982), Berger, Hanweck, and Humphrey (1987), Flannery (1983), Gilligan, Smirlock, and Marshall (1982, 1984), Gilligan and Smirlock (1984), Hancock (1985), Lawrence and Shay (1986), Murray and White (1983), and Mullineaux (1978).

[8]It is assumed that banks operate in competitive input markets, and that they minimize cost for a given level of output.

The cost measure OSE will be less than one if the bank is experiencing increasing returns to scale, since costs will rise proportionally less then output. An OSE value equal to one indicates constant returns to scale, and a value greater than one indicates decreasing returns to scale.[9]

The issue of the existence of economies of scale for the banking firm is of concern in determining appropriate regulatory policy toward bank acquisitions and mergers and in predicting future industry structure. If economies of scale exist in banking, then least cost provision of banking services would be accomplished by having relatively fewer, but larger banks. Competitive market forces will contribute to further consolidation of firms in the banking industry, as larger, more cost efficient firms drive smaller, less efficient firms from the market.

THE DATA

Data from the Functional Cost Analysis program of the Federal Reserve System for the years 1979–86 were used to estimate the cost function parameters. This program is administered to participating financial institutions on a voluntary basis by their regional Federal Reserve bank, and coordinated nationally by the Federal Reserve Bank of New York. This data set has been used widely in previous research, and contains detailed information on the number of accounts of various types that the institution has, as well as their dollar volume. In addition, information on the inputs used in the bank production process, such as the number of officers and employees, their average salaries and benefits, and the number and type of offices the institution has is reported in the FCA program. A functional allocation of some costs across the institution's activities is made, although this allocation by the reporting institution's employees is not used in this study. The advantages of using FCA data are that detailed information on both inputs and outputs is provided in a standardized format and its use enables comparison with previously published research; disadvantages include nonrandom sampling of the nation's financial institutions and the omission of the largest institutions. Another drawback is that the data made available to the public has some information masked to protect the confidentiality of the participating institutions. For this reason, holding company affiliation is not revealed, the state in which the institution is located is unknown, and institutions cannot be tracked from one year to the next. However, detailed information on the various inputs used and the size and number of deposit, loan, and trust accounts is not available from other sources, such as Call Report data.

The vast majority of the roughly fourteen thousand commercial banks in the United States over the time period of this study had total assets comparable in size to the banks in the FCA program. Although the very largest banks are not represented, banks from less than $10 million in total assets to over $2 billion are in the FCA data set. While the FCA data should not be used to draw conclusions about the nation's largest banks, patterns found in the FCA data may well provide insights about

[9]The variance of OSE is calculated as follows: $Var\ (OSE) = \Sigma_i\ Var\ (\alpha_i) + 2\ \Sigma_i\ \Sigma_j\ Cov\ (\alpha_i, \alpha_j)$.

trends affecting the smaller and medium-sized banks which make up over 90 percent of the firms in the U.S. banking industry. Since participation in the FCA program is voluntary, it seems plausible to think that the FCA banks may be more cost conscious, and possibly more efficient, than nonparticipating banks. In a study whose purpose is to investigate possible changes in the cost structure of banking, the nonrandomness of FCA data is not necessarily undesirable. If it is believed that the FCA banks are likely to be more cost efficient than other banks, it may be possible to detect cost function shifts earlier among FCA banks than among other banks, as the FCA banks adapt more rapidly to regulatory and technological changes.

In this study, the banking firm is viewed as a financial intermediary producing various types of loans and investments using labor, capital, and funds as inputs. Some cost studies have excluded the interest costs of funds and focused on operating costs alone. Others have included interest costs, and they are included here following the intermediation approach outlined in Mester (1987a) which contains a discussion of both approaches. Total costs are the sum of labor, capital, and interest costs. Consistent with the intermediation approach, the dollar volume of accounts is the desired output measure for this study. The output categories of investments, total loans, and trust accounts are used.[10]

For the cost function estimation, the prices of the inputs labor, capital, and funds are used. The sum of wages, salaries, and benefits divided by the total number of employees and officers provides an approximation to an annual price of labor. Similarly, the total interest payments divided by the quantity of funds on which interest was paid gives an approximate price of funds. The price of capital is a composite weighted average price of physical and financial capital following the general procedure used by Hancock (1985). It includes occupancy costs divided by the book value of building and equipment for physical capital. The cost of financial capital includes dividends paid on common and preferred stock, and interest on capital notes and debentures.[11]

Each of the above measures give estimates of average, rather than marginal prices. If the marginal differs greatly from the average, this would be a problem. However, to the extent that individual financial institutions are price takers in the market for labor, funds, and capital, the marginal and average prices paid for additional units of input would be similar, so that expansion of output could occur for each firm individually without significantly raising the prices of the necessary inputs. If all institutions were to simultaneously attempt to expand output, the same conclusion might not hold, as pecuniary externalities could be important.

Previous research has found that state branching regulations have an important impact on the firm's cost function. Since this finding was supported in preliminary

[10]For zero levels of output, which occurred for some banks with respect to the trust account output, .01 was substituted for zero. Additional estimates were made where smaller constants were substituted, without materially affecting the empirical results reported in Tables 1 and 2.

[11]Data limitations prevent the inclusion of changes in stock prices which are an important component of the return to investors, and thus may affect the costs of raising capital for banks. An alternative capital price specification following Mester (1987b) was tried in preliminary analyses, but that specification did not change the general pattern of increasing economies of scale reported in Tables 1 and 2.

analyses for this study, separate equations are estimated for banks in unit and branching states.

The number of total offices (full and limited service) the bank has is included in the model for banks in branching states, consistent with previous research which has found this to be an important element affecting cost.

RESULTS

The raw, unedited data from the Functional Cost Analysis data set contain several problems. For example, several banks reported having zero offices, some reported interest payments in excess of total account balances, and others reported negative account balances. The raw data included information on 4,390 banks, which was reduced to 4,277 after screening for the above problems. This results in a sample of from 514 to 735 banks in each year. Estimates were obtained using Zellner's (1962) Seemingly Unrelated Regressions (SUR) technique on a system of equations consisting of the total cost equation and two of the three cost share equations. Estimates of overall economies of scale are constructed from the parameter estimates.[12] Estimates of economies of scope are not calculated in this paper.[13] In calculating the scale estimates, the input prices are held constant at the sample means, and the output levels are allowed to vary. In order to examine economies of scale for banks of different sizes, scale estimates were conducted over a range of asset size categories. The OSE measures were calculated at the output means for each size category. Estimation of a separate model for each of these years allows the comparison of the behavior of the cost function across the time period.

The results of the SUR estimates of overall scale economies for banks in branch banking states are shown in Table 1. Some general patterns can be observed. Scale economies generally diminish as banks get larger within each yearly sample. Statistically significant diseconomies of scale are found for banks at the larger end of the

TABLE 1

OVERALL ECONOMIES OF SCALE BY YEAR: BRANCH BANKING STATES

Total Assets	1979	1980	1981	1982	1983	1984	1985	1986
less than $50	.9504	.9340	.9223	.9433	.9270	.9348	.9151	.9270
	(4.30)	(5.65)	(6.09)	(3.32)	(3.59)	(3.22)	(4.64)	(4.74)
$50–100	.9689	.9613	.9602	.9685	.9300	.9436	.9316	.9370
	(3.34)	(4.23)	(4.01)	(2.33)	(4.59)	(3.70)	(4.83)	(4.96)
$100–200	.9824	.9814	.9904	.9899	.9345	.9509	.9462	.9446
	(2.07)	(2.15)	(1.05)	(0.87)	(4.94)	(3.73)	(4.17)	(4.62)
$200–300	.9922	1.0007	1.0151	1.0073	.9392	.9577	.9573	.9508
	(0.87)	(−0.07)	(−1.46)	(−0.59)	(4.40)	(3.11)	(3.31)	(4.11)
$300–500	1.0046	1.0136	1.0353	1.0234	.9430	.9628	.9668	.9576
	(−0.46)	(−1.29)	(−3.23)	(−1.71)	(3.51)	(2.32)	(2.26)	(3.30)
over $500	1.0253	1.0431	1.0819	1.0541	.9492	.9728	.9871	.9747
	(−1.87)	(−2.96)	(−5.22)	(−2.75)	(2.28)	(1.17)	(0.68)	(1.43)

NOTE: T-statistics in parentheses. T-tests were constructed to test the hypothesis that $OSE = 1$. Total assets are in millions of dollars. OSE values less than 1 indicate economies of scale; values greater than 1 indicate diseconomies of scale.

TABLE 2

OVERALL ECONOMIES OF SCALE BY YEAR: UNIT BANKING STATES

Total Assets	1979	1980	1981	1982	1983	1984	1985	1986
less than $50	.9616	.9759	.9681	.8904	.8849	.9398	.9736	.8504
	(3.04)	(1.61)	(1.70)	(5.77)	(5.06)	(2.16)	(0.87)	(5.00)
$50–100	.9806	.9853	.9854	.9183	.9066	.9633	.9894	.9005
	(1.94)	(1.48)	(0.98)	(5.62)	(5.48)	(2.01)	(0.52)	(4.87)
$100–200	.9925	.9925	.9928	.9451	.9252	.9806	.9948	.9392
	(0.74)	(0.73)	(0.52)	(3.82)	(4.65)	(1.02)	(0.26)	(2.27)
$200–300	1.0015	1.0013	1.0006	.9718	.9431	.9882	1.0017	.9692
	(−0.12)	(−0.10)	(−0.04)	(1.56)	(3.20)	(0.51)	(−0.07)	(0.88)
$300–500	1.0113	1.0000	1.0134	.9870	.9516	1.0014	1.0106	.9875
	(−0.79)	(−0.00)	(−0.65)	(0.60)	(2.28)	(−0.05)	(−0.33)	(0.29)
over $500	1.0246	1.0086	1.0230	1.0031	.9803	1.0204	1.0111	1.0288
	(−1.35)	(−0.39)	(−0.87)	(−0.11)	(0.69)	(−0.50)	(−0.25)	(−0.51)

NOTE: T-statistics in parentheses. T-tests were constructed to test the hypothesis that $OSE = 1$. Total assets are in millions of dollars. OSE values less than 1 indicate economies of scale; values greater than 1 indicate diseconomies of scale.

sample size in the early years, but not after 1982. Statistically significant economies of scale are found for the smaller size categories (less than $100 million in total assets) of banks in all years. The range over which significant economies of scale are found increases up to $500 million in total assets in 1983–86, and extends over $500 million in 1983. The overall pattern appears to be one of increasing economies of scale over the entire time period.

The scale economies estimates for unit banking states are shown in Table 2. The general pattern is similar to that for branching states, although the pattern is not as strong. Statistically significant economies of scale are found for banks with less than $50 million in total assets in 1979, 1982, 1983, 1984, and 1986. The extent of scale economies increases in most of the later years, with statistically significant economies of scale found for banks through the $100–$200 million size range in 1982, 1983, and 1986.

While the general pattern of increasing economies of scale is found for banks in both unit and branching states, the pattern appears stronger for the branching state banks. One possible explanation for this result is the difference in the size of the branching and unit state samples. The branch state sample contains roughly seventy to two hundred more banks than the unit state sample in each year. The larger number of observations should result in cost function parameter estimates that are close to the "true" underlying functional relationships. There also was a greater proportional sample size decline in the unit state sample which makes any shift in the underlying cost function relationships less likely to be detected by statistical investigation. Another possibility is that the branching restrictions effectively constrain efficient bank structure, and this constraint prevents larger banks from exploit-

[12]Parameter estimates are available from the author upon request.

[13]The translog functional form is not well suited to the calculation of economies of scope. Scope economies were calculated from a set of hybrid translog parameter estimates. As a referee has pointed out, the scope estimates are sensitive to the choice of λ in the Box-Cox metric used in the hybrid translog model. Rather than fully investigate this issue, this paper focuses on scale economies.

ing scale economies which could be realized from operating a branching organization.

POOLED MODEL RESULTS AND COST SHIFTS

An alternative to using the results from the yearly regressions to study possible shifts in production technology across the 1979–1986 time period is to pool the annual data, and compare the results across pools. While pooling these data raises a number of issues, it was done to provide a comparison of results with the annual analyses.[14]

In the pooled data for banks in both unit and branching states, the asset size levels through which significant economies of scale were found and those where diseconomies of scale were realized both increased over the time periods. This general pattern is the same as that exhibited in the annual regressions, although, as expected, the pattern from the pooled data appears to be somewhat stronger.

SHIFTS ACROSS PERIODS

Two methods were used to examine the possibility of structural change in the cost function across time periods. The first was to construct a joint test that all parameters in each model were the same across time periods. The hypothesis that all parameters are the same in each period is rejected at the 1 percent level in each case, indicating structural changes in the cost function.

The second method was to focus on the estimated scale economies measures, and test whether those were the same across periods. For the respective sample mean banks in both branch and unit states, there were statistically significant increases in economies of scale between the 1979–82 and 1983–86 time periods. This is also true between the 1979–81 and 1984–86 time periods. In general, the overall trend again appears to be one of increasing economies of scale.

SUMMARY AND CONCLUSIONS

The behavior of the cost function for a sample of commercial banks over the years 1979–1986 has been investigated. For the years prior to 1982, little evidence to

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[14]These issues include but are not limited to the stability of the sample size across years, the actual banks in the sample in each year, the geographic distribution of the sample in each year, and the effects of inflation. The pooled analysis was done with two pooling periods: 1979–82 and 1983–86. Another analysis was done with three pooling periods: 1979–81, 1982–83, and 1984–86. The pooled analyses were conducted after adjusting all output quantities, expenditures and prices to constant 1982 prices, and repeated without making any adjustments. These analyses are available on request from the author. There are only minor differences in the OSE estimates from the adjusted and unadjusted analyses. The primary point, that there are statistically significant increases in economies of scale over the 1979–1986 time period, is unaffected by adjusting the regression variables for the effects of inflation. This conclusion is also true for the annual analyses.

suggest that economies of scale exist beyond small levels of output was found. This result is consistent with most of the empirical literature on bank costs. In contrast to the earlier years and previous literature, statistically significant scale economies were found in the later years for banks in branching and unit states. The degree of scale economies also increased over the 1979–86 time period. These results suggest that the effect of recent regulatory and technological changes may have been to give larger banks a cost advantage over that which existed in previous years. These results also indicate that there may be increased cost pressure for smaller banks to become larger, either through mergers and acquisitions or though internal growth. This may lead to further consolidation pressures within the industry, and reductions in the overall number of banking firms. Virtually all of the previous cost studies concluded with the caution that the results they had found might not apply as the deregulation process continued, and further technological developments occurred. The results found in the present study indicate that those authors were correct.

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