FINITE MIXTURE ESTIMATION OF MULTIPRODUCT COST FUNCTIONS

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Abstract—This paper presents a technique of cost function estimation, based on the theory of finite mixture distributions, which allows for the simultaneous existence of multiple technologies of production when the researcher does not know which observations correspond to which technologies. The finite mixture technique provides estimates of the proportions of firms using the various technologies, facilitates comparisons between technologies, and preserves the traditional interpretations of cost estimation. After describing the mixture procedure, the technique is illustrated on a large sample of savings and loan associations, and it is concluded that this industry exhibits multiple technologies of production.

I. Introduction

SINCE the pioneering work on flexible functional forms of Diewert (1973), Christensen, Jorgenson, and Lau (1971), Färe and Jansson (1975), and others, the estimation of multiproduct cost functions has become one of the primary tools of applied economics. Usually, firm-level observations on costs and outputs for a particular industry are pooled as a cross-section, and the resulting regression equations are interpreted as approximations to some true, underlying cost function shared by all firms in the sample. Because many flexible form cost functions allow evaluation of the direction and extent of scale and scope economies, the results of cost estimations play prominent roles in discussions of industrial, antitrust, and regulatory policy.

The widespread use of cost estimation has led to considerable debate about its merits. Issues raised include the "approximating power" of the translog and other flexible forms, the relationships and inconsistencies between econometric and engineering cost functions, and the role of technical efficiency in cost analysis.[1] Although this debate has been wide-ranging, most analyses of these issues are based on the same fundamental assumption: all (sample) firms use the same technology, and hence face the same (long-run) cost function. Any deviation between a particular firm's costs and those implied by the common cost relationship is attributed to random factors or inefficiency, yet the cost function that defines average or frontier efficient performance is assumed to apply to all firms.

There are reasons to believe, however, that all firms in a given industry may not share a common technology, particularly when innovations or regulatory changes occur. As Reingamun notes, "an important empirical observation regarding the adoption of innovations is that adoption is typically delayed and that firms do not adopt an innovation simultaneously."[2] In his pioneering study of the diffusion of important innovations within industries, Mansfield found that "... the diffusion of a new technique is generally a rather slow process," and that some innovations had not been adopted by all firms in several industries he studied even a decade after their introductions.[3] While economic analyses of the diffusion of innovations have historically focused on capital-embodied technology, it is almost certainly incorrect to assume that all important innovations reside in machinery: sophisticated managerial practices like "just in time" inventory control and strategic management planning both influence costs and are unlikely to be simultaneously adopted by everyone. Finally, regulation and political pressure can substantially influence the technological choices of firms: examples include the certification of nuclear power plants and product-line restrictions imposed on financial institutions. There are thus a number of reasons why the assumption that all firms share a com-

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[1] The flexibility of the translog is evaluated by Gallant (1981). The relationships between engineering and econometric production functions are discussed by Marsden, Pingry, and Whinston (1974) and Berndt and Wood (1979). The role of technical efficiency is discussed by Färe, Grosskopf, and Lovell (1985), Färe (1988), and Forsund and Jansen (1977).

[2] Reingamun (1989), p. 383.

[3] Mansfield (1968a), p. 136.

mon technology may be incorrect for many industries.

The simultaneous existence of multiple technologies of production can pose a serious problem for traditional cost function estimation. If a sample includes observations on firms using different technologies, pooling the data in a single regression procedure is likely to produce misleading results. Since the single-technology restriction is a specification error in these cases, the estimated cost function obtained may not be quantitatively (e.g., in coefficient magnitudes), nor qualitatively (e.g., in the implied presence or absence of scale effects) similar to any of the true, underlying cost relationships.[4] If the researcher knew which observations corresponded to which technologies, then this problem could be avoided by estimating several cost functions. Unfortunately, this kind of information is usually not available. When "technology" is broadly interpreted to include "know-how," it is not clear that the necessary information could ever be obtained. Without such information, however, traditional cost function estimation may be unreliable.

This paper presents a technique, based on the statistical theory of finite mixture distributions, which allows the estimation of cost functions based on different technologies when the researcher does not know which observations correspond to which technologies. The mixture technique estimates the proportions of firms using the technologies, allows limited testing for the number of technologies concurrently in use, and facilitates quantitative and qualitative comparisons between technologies.

In order to illustrate the mixture technique, we estimate both translog and flexible fixed cost quadratic cost functions for a large sample of solvent savings and loan companies in the United States, using both the traditional pooled and mixture techniques. We find both substantial support for the claim that this industry is characterized by multiple technologies of production, and evidence that traditional, pooled estimation can produce misleading results.

The paper is divided into six sections. Section II outlines some theory of mixture distributions and relates this theory to cost estimation. Section III describes the data set and cost function specifications selected to illustrate the methodology. Section IV presents and discusses the estimation results, while section V examines the relationship between the mixture methodology and frontier estimation procedures. A conclusion which summarizes the results and suggests extensions completes the paper.

II. Finite Mixture Theory and Cost Estimation

The theory of finite mixture distributions is a venerable branch of statistics: early work includes that of Newcomb (1886) and Pearson (1894). The primary uses of finite mixture models are in robust estimation and in circumstances in which sampling occurs from a population composed of several subpopulations, although the specific subpopulation from which any given observation is drawn is unobserved.[5] While finite mixture models are often seen in botany, biology, geology, and zoology, their use in analyzing economic problems has been limited. Economic applications, which often stress the relationship between mixture techniques and switching regressions, include wage regressions (Quandt and Ramsey (1978)), housing sales (Quandt (1972)), and numbers of purchases (Paull (1978)).

The marginal density function for a variable with a finite mixture distribution is a convex combination of marginal density functions. Formally, if a random variable ϵ on the real line R has a marginal distribution $h(\epsilon)$ given by

$$h(\epsilon) = \sum_{j=1}^{k} \lambda_j f_j(\epsilon) \tag{1}$$

where

$$\lambda_j > 0, \forall j$$

$$\sum_{j=1}^{K} \lambda_j = 1$$

$$f_j(\epsilon) > 0, \quad j = 1, 2, \ldots, K$$

and

$$\int_{-\infty}^{\infty} f_j(\epsilon)\, d\epsilon = 1, \qquad j = 1, 2, \ldots, K$$

[4] Pooled estimation may be subject to substantial bias since the literature on firm adoption of innovations identifies such things as firm size as important. See David (1969) for a discussion of the diffusion of new technologies.

[5] A discussion of various uses of mixture models and a catalog of published research is in Titterington, Smith, and Makov (1985).

then ϵ is said to have a K-component finite mixture distribution with mixing weights $\lambda_1, \lambda_2, \ldots, \lambda_K$ and component densities $f_1(\cdot), f_2(\cdot), \ldots, f_K(\cdot)$. While the component densities are usually assumed to be members of the same parametric family, this kinship is not required.

The most commonly discussed mixture model is that of two univariate normal distributions. Fundamental identifiability results for this and other more general models are available in Yakowitz (1969), Teicher (1963), and Chandra (1977). Extended discussions of the theory and estimation of mixture models are given by Titterington, Smith, and Makov (1985) and Everitt and Hand (1981).

The application of the finite mixture methodology to the estimation of cost functions is an extension of traditional cost estimation. In the usual estimation of cost relationships, costs C are assumed to be some function of outputs Y, input prices P, a vector of parameters θ, and a zero-mean, normally distributed error ϵ:

$$C = g(Y, P; \theta) + \epsilon. \tag{2}$$

Estimates of θ are then obtained by least squares or other techniques.

In estimating the normal two-component finite mixture cost function model, two cost functions, $g_1(Y, P; \theta_1)$ and $g_2(Y, P; \theta_2)$, are assumed to exist. The probability that a randomly selected observation on costs C is generated by the process $C = g_1(Y, P; \theta_1) + \epsilon_1$ is denoted λ, while the probability costs are generated by the process $C = g_2(Y, P; \theta_2) + \epsilon_2$ is $(1 - \lambda)$, where ϵ_1 and ϵ_2 are stochastically independent, zero-mean spherically normally distributed random variables with variances σ_1^2 and σ_2^2, respectively, and the mixing weight λ satisfies $0 \leq \lambda \leq 1$. Note that, when $\lambda = 0$ or $\lambda = 1$, the two component mixture model collapses to the single cost function model given by equation (2).

The likelihood function L for the two component normal mixture cost model is given by

$$L = \prod_i \left[\lambda\phi_1(C_i - g_1(Y_i, P_i; \theta_1)) + (1 - \lambda)\phi_2(C_i - g_2(Y_i, P_i; \theta_2))\right] \tag{3}$$

and where the subscript i identifies observations, and $\phi_j(\cdot)$, $j = 1, 2$, are normal marginal density functions with variances σ_1^2 and σ_2^2, respectively. This likelihood function can be maximized over θ_1, θ_2, σ_1^2, σ_2^2, and λ using the E-M algorithm in the manner suggested by Hartley (1978).[6]

III. Data and Model Specification

In order to illustrate the finite mixture technique, we estimated cost functions for savings and loan associations operating in the United States at the end of 1988. Recent important regulatory changes in this industry motivated our interest. One impact of deregulation was to reduce the product line restrictions under which S & L's operated. Such significant changes as allowing production of new outputs suggest the possibility that new techniques of management and production, i.e., a new technology, may emerge through time, replacing forms of organizing operations that were common prior to deregulation. Thus the savings and loan industry may be a good example of an industry using multiple technologies for which the finite mixture procedure would be appropriate.

Raw data on savings and loan associations came from the Office of Thrift Supervision's Thrift Financial Report (TFR) tape for December 1988. The data were screened to eliminate institutions reported as having zero or negative total assets or deposits, as well as those reporting zero or negative interest or operating costs. Further, we eliminated from the data set all firms that were insolvent on a "regulatory capital" basis in 1988, so that all firms used in estimation were viable during the sample period.[7]

The TFR tape provided data on loan volumes, interest, and operating and non-operating costs. Average interest and capital costs were calculated for each firm individually using techniques common in financial firm cost function estimation

[6] Our experience suggests that the E-M algorithm is the best choice for mixture estimation.

[7] This screening assures that a finding of the apparent existence of two technologies will not merely represent cost differences between solvent and insolvent institutions. Additionally, issues of output homogeneity arise when comparing costs of viable and insolvent S & L's, although the relevant issue is not whether some loans are repaid and others are not: it is not necessarily cheaper to produce something that is stolen than something that is sold. As a practical matter, eliminating insolvent firms had little effect on estimation results.

studies.[8] Wage data for workers in the finance, insurance, and real estate industries by state for 1988 were obtained from the Bureau of Labor Statistics.[9]

Our selection of outputs and input prices was dictated by previous cost function studies of financial firms. Outputs used in our estimations are the dollar volumes of consumer loans, commercial loans, and mortgages. Inputs are deposits and borrowed money, labor, and capital. We thus model the savings and loan association as a financial intermediary which takes deposits and other inputs and creates loans. For a detailed, recent survey of this literature, see Clark (1988).

In specifying the functional forms to be used in estimation, we strived for two goals. First, the cost functions specified had to exhibit sufficient flexibility so that results from the mixture procedure would not merely represent the inability of the functional form selected to approximate the cost surface. Second, the cost forms selected had to be well known and applicable to cost function estimation for financial enterprises. For these reasons we selected both the translog and flexible fixed cost quadratic cost functions for both traditional pooled and finite mixture estimation.

The translog specification used in our estimations is given by

$$\begin{aligned}\ln C = \alpha_0 &+ \Sigma_i \alpha_i F_i + \Sigma_i \beta_i \ln Y_i \\ &+ \tfrac{1}{2}\Sigma_i \Sigma_j \delta_{ij} \ln Y_i \ln Y_j \\ &+ \Sigma_i \tau_i \ln P_i + \tfrac{1}{2}\Sigma_i \Sigma_j \eta_{ij} \ln P_i \ln P_j \\ &+ \Sigma_i \Sigma_j \gamma_{ij} \ln Y_i \ln P_j + \epsilon \qquad (4)\end{aligned}$$

where C is cost, Y is a vector of output quantities with i^{th} element Y_i, P is a vector of input prices with i^{th} element P_i, and F is a vector of dummy variables such that $F_i = 1$ if $Y_i > 0$ and $F_i = 0$ otherwise.[10] The α's, β's, δ's, τ's, η's, and γ's are parameters to be estimated, and ϵ is the error term. Linear homogeneity in input prices requires

$$\begin{aligned}&\Sigma_i \tau_i = 1 \\ &\Sigma_j \eta_{ij} = 0, \forall\, i \\ &\Sigma_j \gamma_{ij} = 0, \forall\, i \qquad (5)\end{aligned}$$

and we impose these conditions in estimation.

The flexible fixed cost quadratic cost function (FFC), suggested by Baumol, Panzar, and Willig (1982), has been applied by Friedlaender, Winston, and Wang (1983), Mayo (1984), Cohn, Rhine, and Santos (1989) and others using various input price specifications. The FFC is easy to estimate, and allows straightforward evaluation of scale and scope economies. The FFC is given by

$$\begin{aligned}C = \alpha_0 &+ \Sigma_i \alpha_i F_i + \Sigma_i \beta_i Y_i \\ &+ \tfrac{1}{2}\Sigma_i \Sigma_j \delta_{ij} Y_i Y_j \\ &+ \Sigma_i \tau_i \ln P_i + \tfrac{1}{2}\Sigma_i \Sigma_j \eta_{ij} \ln P_i \ln P_j \\ &+ \Sigma_i \Sigma_j \gamma_{ij} Y_i \ln P_j + \epsilon \qquad (6)\end{aligned}$$

where the variables are as in (4). Because of the form of the FFC model, linear homogeneity in input prices cannot be imposed.[11]

IV. Estimation and Results

Descriptive statistics and definitions for the variables used in this study appear in table 1. As the table makes clear, sample firms exhibit substantial variations in the sizes and scopes of their operations. While all sample firms offered home mortgage loans, and nearly all offered consumer loans, only about half of sample firms offered commercial loans during the sample period. Borrowed money, which constitutes approximately three-fourths of typical S & L costs for sample firms, was obtained at an average cost of about seven cents per dollar borrowed in 1988.

The FFC and translog cost models were estimated using both finite mixture and traditional techniques by maximum likelihood using the E-M algorithm. The parameter estimates, t-scores, and mixing weights for the pooled and finite mixture translog models appear in table 2, while table 3

[8] The price of funds was calculated by subtracting fees charged from interest paid on funds, and dividing that amount by the total volume of funds, a practice consistent with other studies in this area (Clark (1988)). Capital costs were calculated using the procedure outlined in Mester (1987).

[9] This compromise was dictated by data availability, but may not be a problem since the costs of funds alone constitute about three-fourths of total costs for the average S & L in the sample.

[10] Consistent with other studies in this area, firms reporting zero levels of some outputs are treated in translog estimation as if they had output levels of 1¢. While this assumption may not be innocuous, our primary purpose is to compare the pooled and mixture results, rather than to place great emphasis on the results themselves. Additionally, mixture estimation including Box-Cox transformation parameters is problematic.

[11] For discussions of this issue, see Friedlaender, Winston, and Wang (1983) and Cohn, Rhine, and Santos (1989).

TABLE 1.—DESCRIPTIVE STATISTICS FOR REGRESSION VARIABLES

Variable Name	Description	Mean	Standard Deviation
TOTCOST	Total Interest, Compensation, and Capital costs	7,984.5	31822.8
COMMFIX	= 1 if *COMMLOAN* > 0, = 0 otherwise	0.51	0.50
CONSFIX	= 1 if *CONSLOAN* > 0, = 0 otherwise	0.99	0.10
MORTGAGE	Net Mortgage Loans, Contracts and Pass through Securities	281,860.0	1,185,444.4
COMMLOAN	Secured and unsecured non-mortgage commercial loans	6,515.6	58783.5
CONSLOAN	Non-mortgage consumer loans	11,224.4	55228.9
WAGE	Average hourly wage; Finance, Insurance and Real Estate Industry	12.19	2.17
PFUNDS	Net interest price of funds; annual percentage basis	7.22%	0.7%
PCAP	Physical capital costs as a percentage of total deposits; annual basis	0.48%	0.3%

Note: All figures are expressed in thousands of dollars, except *COMMFIX*, *CONSFIX*, *WAGE*, *PFUNDS* and *PCAP*. Data were obtained from the Office of Thrift Supervision, Thrift Financial Report, December 1988, and the Bureau of Labor Statistics. The final sample contained observations from 2092 savings and loan associations.

presents equivalent information for the FFC models.[12]

Referring to table 2, we note first that, for the translog model, the finite mixture procedure suggests the presence of two technologies, of which the first, arbitrarily denoted "Mixture 1," is the relevant cost relationship for approximately 80% of sample firms.[13] While formally testing for the presence of a mixture is not straightforward, the approximate test suggested by Wolfe (1971) can be applied.[14] Performing the necessary calculation, we obtain a corrected Chi-square value of 805.9, allowing us to reject at the 1% level of significance the claim that the sample is characterized by a single cost relationship.

Table 2 also illustrates the quantitative differences in underlying production technologies implied by the traditional and finite mixture estimations. In seven cases out of thirty, the estimated coefficients in the pooled model fail to lie between the two corresponding estimated coefficients of the mixture model.[15] Further, the magnitudes of estimated coefficients differ substantially between models, and in five cases there are sign differences in significant coefficients between the pooled and mixture regression relationships.

Similar quantitative differences for the FFC model are illustrated in table 3. In this model specification, the dominant cost relationship (again arbitrarily denoted Mixture 1) is estimated to be applicable to approximately 86% of sample firms. Repeating Wolfe's test for the presence of a mixture, we obtain a Chi-square test statistic value of 6566.2, again allowing strong rejection of the hypothesis that there is a single technology of production. Additionally, the pooled model's co-

[12] The high t scores for many coefficients are typical in flexible form cost function estimation.

[13] A number of estimations, utilizing various cost specifications and different definitions for input prices are not reported here, all uniformly produced mixing weights between 75% and 86%.

[14] The hypothesis $\lambda = 0$ or $\lambda = 1$ involves a parameter value on the boundary of the parameter space, violating necessary regularity conditions. Evidence from simulations led Wolfe (1971) to suggest a modified likelihood ratio test. For details, see Wolfe (1971) or Everitt and Hand (1981).

[15] There is, however, no mathematical requirement that the coefficients be so ordered.

TABLE 2.—TRANSLOG COST FUNCTION PARAMETER ESTIMATES

Variable	Pooled	Mixture 1	Mixture 2
INTERCEPT	−0.03700[a]	−0.04714[a]	0.11861[a]
	(−2.712)	(−5.967)	(7.021)
COMMFIX	0.00636	0.00825	−0.07264[a]
	(0.261)	(0.598)	(−2.352)
CONSFIX	0.03215	0.01218	0.05164[a]
	(1.771)	(1.085)	(2.185)
MORTGAGE	0.88355[a]	0.88861[a]	0.86711[a]
	(114.493)	(205.090)	(102.874)
COMMLOAN	0.04723[a]	0.04001[a]	0.09133[a]
	(6.940)	(11.080)	(10.881)
CONSLOAN	0.04593[a]	0.05703[a]	0.02177[a]
	(6.312)	(13.562)	(2.953)
MORTGAGE^2	0.04781[a]	0.04465[a]	0.05397[a]
	(9.275)	(14.034)	(11.278)
*MORTGAGE * COMMLOAN*	−0.00704[a]	−0.00486[a]	−0.01664[a]
	(−3.145)	(−4.594)	(−6.197)
*MORTGAGE * CONSLOAN*	−0.02140[a]	−0.03103[a]	−0.00357
	(−6.107)	(−14.167)	(−1.467)
COMMLOAN^2	0.01306[a]	0.01036[a]	0.03015[a]
	(5.573)	(9.785)	(9.693)
*COMMLOAN * CONSLOAN*	−0.00265	−0.00232[a]	−0.00315
	(−1.441)	(−2.196)	(−1.73)
CONSLOAN^2	0.02033[a]	0.02857[a]	0.00089
	(6.011)	(13.062)	(0.356)
WAGE	0.05252	0.08012[a]	0.01736
	(1.253)	(3.338)	(0.458)
PFUND	0.89088[a]	0.82993[a]	1.01899[a]
	(20.970)	(34.484)	(27.269)
PCAP	0.05659[a]	0.8994[a]	−0.03625[a]
	(3.450)	(9.420)	(−2.446)
WAGE^2	0.29889[a]	0.27192[a]	−0.32301[a]
	(4.104)	(7.155)	(−9.454)
PFUND^2	0.73858[a]	0.38182[a]	0.33319[a]
	(8.025)	(6.347)	(6.429)
PCAP^2	0.10883[a]	0.05683[a]	0.26553[a]
	(6.710)	(6.766)	(18.915)
*WAGE * PFUND*	−0.46432[a]	−0.29845[a]	0.12767[a]
	(−6.002)	(−6.439)	(2.288)
*WAGE * PCAP*	0.16542[a]	0.02653	0.19534[a]
	(4.987)	(1.342)	(6.690)
*PFUND * PCAP*	−0.27426[a]	−0.10124[a]	−0.36465[a]
	(−8.379)	(−4.975)	(−18.857)
*WAGE * MORTGAGE*	−0.02008	−0.01890	0.06193[a]
	(−1.019)	(−1.487)	(4.050)
*WAGE * COMMLOAN*	0.00766	0.01041	−0.04006[a]
	(0.631)	(1.537)	(−4.393)
*WAGE * CONSLOAN*	−0.04194[a]	−0.00086	−0.10098[a]
	(−2.547)	(−0.080)	(−8.227)
*PFUND * MORTGAGE*	0.03626	0.02027	−0.07634[a]
	(1.871)	(1.617)	(−5.257)
*PFUND * COMMLOAN*	0.00083	−0.00964	0.06318[a]
	(0.067)	(−1.373)	(6.671)
*PFUND * CONSLOAN*	0.04031[a]	0.00264	0.11649[a]
	(2.439)	(0.246)	(9.602)
*PCAP * MORTGAGE*	−0.01618[a]	−0.00137	0.01440[a]
	(−2.313)	(0.350)	(2.752)
*PCAP * COMMLOAN*	−0.00850	−0.00077	−0.92317[a]
	(−1.754)	(−0.286)	(−5.533)
*PCAP * CONSLOAN*	0.00162	−0.00178	−0.01550[a]
	(0.294)	(−0.578)	(−2.989)
λ	—	0.80209	0.19791
σ	0.1764	0.1005	0.2365

Note All variables in logs as appropriate. Asymptotic t-statistics in parentheses.
[a] Significant at the 5% level.

TABLE 3.—FFC QUADRATIC COST FUNCTION PARAMETER ESTIMATES

Variable	Pooled	Mixture 1	Mixture 2
INTERCEPT	0.000191	−0.008182	0.215602
	(0.002)	(−0.819)	(0.079)
COMMFIX	0.007791	−0.000572	0.065105
	(0.409)	(−0.374)	(1.985)
CONSFIX	0.007384	0.011932	−0.36045
	(0.084)	(1.196)	(−0.013)
MORTGAGE	0.851467[a]	0.855646[a]	0.779106[a]
	(94.624)	(643.822)	(112.631)
COMMLOAN	0.087177[a]	0.040133[a]	0.060031[a]
	(17.857)	(59.353)	(14.642)
CONSLOAN	0.037806[a]	0.046795[a]	0.54052[a]
	(6.168)	(59.801)	(19.277)
$MORTGAGE^2$	0.000951[a]	−0.000907[a]	0.003186[a]
	(−4.164)	(−10.351)	(−8.441)
*MORTGAGE * COMMLOAN*	0.000373[a]	−0.001671[a]	0.012041[a]
	(2.658)	(−48.759)	(27.557)
*MORTGAGE * CONSLOAN*	−0.000763[a]	−0.000176[a]	0.002708[a]
	(2.651)	(−2.347)	(6.792)
$COMMLOAN^2$	−0.000391[a]	0.000364[a]	−0.009794[a]
	(−3.812)	(18.559)	(−41.367)
*COMMLOAN * CONSLOAN*	−0.000181	0.001714[a]	0.005332[a]
	(−1.666)	(39.394)	(31.280)
$CONSLOAN^2$	−0.000546[a]	0.000011	−0.004953[a]
	(−2.206)	(0.231)	(−18.336)
WAGE	0.097131	0.005416[a]	−0.106687
	(1.478)	(0.930)	(−0.615)
PFUND	0.094150	−0.55073[a]	−0.315665
	(0.827)	(−6.273)	(−1.297)
PCAP	0.033775	0.009140[a]	0.103407[a]
	(1.725)	(5.367)	(1.982)
$WAGE^2$	0.024077	0.027877	−0.907685
	(0.050)	(0.739)	(−0.761)
$PFUND^2$	0.810156[a]	0.151903[a]	−0.615216
	(2.974)	(7.360)	(−1.189)
$PCAP^2$	0.033684	0.006603[a]	0.187883[a]
	(0.955)	(2.177)	(2.941)
*WAGE * PFUND*	−0.153712	−0.117038[a]	−0.956687
	(−0.278)	(−2.831)	(−1.038)
*WAGE * PCAP*	0.055608	0.018626[a]	−0.538010[a]
	(0.524)	(2.217)	(−2.488)
*PFUND * PCAP*	0.330944[a]	0.037326[a]	0.650128[a]
	(3.460)	(6.118)	(3.423)
*WAGE * MORTGAGE*	−0.073535[a]	0.035115[a]	−0.061797[a]
	(−2.115)	(5.788)	(−3.589)
*WAGE * COMMLOAN*	−0.014386	−0.045842[a]	−0.074195[a]
	(−0.501)	(−7.313)	(−3.670)
*WAGE * CONSLOAN*	−0.036792	0.007400	0.099706[a]
	(−1.468)	(1.852)	(−6.545)
*PFUND * MORTGAGE*	0.839108[a]	1.065180[a]	1.802360[a]
	(17.086)	(161.342)	(40.027)
*PFUND * COMMLOAN*	−0.021183	0.023356[a]	0.011817
	(−0.583)	(3.479)	(0.416)
*PFUND * CONSLOAN*	0.358152[a]	−0.151428[a]	−0.198119[a]
	(5.854)	(−16.671)	(−7.097)
*PCAP * MORTGAGE*	−0.005941	0.008165[a]	0.081607[a]
	(−0.534)	(6.450)	(13.458)
*PCAP * COMMLOAN*	−0.075776[a]	0.028298[a]	−0.188324[a]
	(−9.047)	(21.182)	(−42.208)
*PCAP * CONSLOAN*	0.032744[a]	0.001501	−0.008412[a]
	(3.991)	(0.838)	(−2.522)
λ	—	0.85543	0.14457
σ	0.4033	0.0315	0.84505

Note: Asymptotic *t*-values are in parentheses.
[a] Significant at the 5% level.

TABLE 4.—SCALE AND SCOPE MEASURES FOR FFC QUADRATIC COST FUNCTION

Percentage of Means	Overall Scale	Product Specific Scale			Overall Scope	Product Specific Scope		
		Mort	Comm	Cons		Mort	Comm	Cons
				Pooled Model				
50%	1.032[a]	1.001[a]	1.182	1.401	0.001	0.000	0.001	0.000
	(2.73)	(4.29)	(0.41)	(0.09)	(0.00)	(0.00)	(0.00)	(0.00)
75%	1.022	1.001[a]	1.124	1.274	0.000	−0.000	0.001	−0.000
	(1.60)	(4.29)	(0.42)	(0.09)	(0.00)	(−0.00)	(0.01)	(−0.00)
100%	1.018	1.001[a]	1.095	1.213	0.000	−0.000	0.001	−0.000
	(1.08)	(4.28)	(0.43)	(0.00)	(0.00)	(−0.00)	(0.01)	(−0.00)
200%	1.011	1.002[a]	1.055	1.130	−0.000	−0.001	0.001	−0.001
	(0.37)	(4.28)	(0.49)	(0.11)	(−0.00)	(−0.02)	(0.03)	(−0.02)
300%	1.011	1.003[a]	1.045	1.113	−0.001	−0.001	0.002	−0.002
	(0.19)	(4.27)	(0.59)	(0.14)	(−0.01)	(−0.04)	(0.06)	(−0.06)
				Mixture Model 1				
50%	1.007[a]	1.001[a]	0.967	1.502	−0.034	−0.016	−0.017	−0.018
	(5.32)	(10.40)	(−0.43)	(1.20)	(−0.82)	(−0.77)	(−0.82)	(−0.86)
75%	1.005[a]	1.001[a]	0.975	1.332	−0.023.	−0.010	−0.012	−0.013
	(3.75)	(10.40)	(−0.51)	(1.20)	(−0.82)	(−0.71)	(−0.82)	(−0.91)
100%	1.004[a]	1.001[a]	0.977	1.247	−0.017	−0.007	−0.009	−0.010
	(3.05)	(10.39)	(−0.61)	(1.19)	(−0.81)	(−0.63)	(−0.82)	(−0.97)
200%	1.003	1.002[a]	0.976	1.119	−0.008	−0.000	−0.004	−0.008
	(1.20)	(10.38)	(−1.33)	(1.19)	(−0.79)	(−0.08)	(−0.84)	(−1.44)
300%	1.003	1.003[a]	0.970[a]	1.077	−0.005	0.003	−0.003	−0.008[a]
	(0.05)	(10.36)	(−2.53)	(1.18)	(−0.76)	(0.84)	(−0.86)	(−2.21)
				Mixture Model 2				
50%	1.545[a]	1.002[a]	3.293	−0.311	0.616	0.306	0.305	0.309
	(20.51)	(8.90)	(1.94)	(−0.01)	(0.08)	(0.08)	(0.08)	(0.08)
75%	1.362[a]	1.003[a]	2.613[a]	0.158	0.459	0.226	0.225	0.230
	(11.45)	(8.88)	(2.02)	(−0.01)	(0.08)	(0.08)	(0.08)	(0.08)
100%	1.270[a]	1.004[a]	2.295[a]	0.404	0.361	0.176	0.174	0.182
	(7.32)	(8.86)	(2.14)	(−0.01)	(0.07)	(0.07)	(0.07)	(0.08)
200%	1.131[a]	1.008[a]	1.938[a]	0.839	0.172	0.077	0.072	0.090
	(2.35)	(8.78)	(2.92)	(−0.01)	(0.06)	(0.06)	(0.05)	(0.07)
300%	1.083	1.012[a]	1.957[a]	1.059	0.085	0.028	0.020	0.049
	(1.18)	(8.70)	(4.14)	(0.00)	(0.05)	(0.03)	(0.02)	(0.05)

Note: Asymptotic t-statistics are calculated using the procedure of Mester (1987), and appear in parentheses For scale measures, the null hypothesis is constant returns (a measure equal to 1), while for scope measures, the null hypothesis is no scope effects (a scope measure of 0).
[a] Significant at the 5% level.

efficients fail to lie between those of the mixture model in 16 of 30 cases.

The importance of the differences in estimated parameters exhibited in tables 2 and 3 can be clearly illustrated by calculating scale and scope economy measures using the various estimated regression relationships and comparing the results. This is done for the results of the FFC model in table 4. The cost measures are not calculated for the translog model because of the well-known difficulties in calculating scope economies with this functional form (Berger, Hanweck, and Humphrey (1987)). The scale and scope economy measures are constructed by the widely-used procedures of Baumol, Panzar, and Willig (1982): an overall or product-specific scale measurement of 1 indicates constant returns, less than 1, decreasing returns, and greater than 1, increasing returns.[16] Similarly, overall or product-specific scope measures greater than zero indicate positive scope economies, zero values suggest no scope effects, and negative values imply scope diseconomies. These measures are calculated and presented in table 4 for selected scalings of the sample mean output vector.

The three technologies represented by the pooled and mixture results exhibit important dif-

[16] See Baumol, Panzar, and Willig (1982) for an extensive exploration of these measures of economies of scope and scale.

ferences. While all three estimated technologies have statistically significant overall scale economies at low to moderate levels of outputs, the mixture results suggest that, for 86% of sample firms, cost advantages obtainable by output expansion are less than one-third as large as the pooled results would suggest. Further, a minority of firms utilize a technology that exhibits very large (though diminishing) statistically significant returns to scale at low to moderate output levels. Differences in the magnitudes of product-specific scale effects for mortgage loans at high output levels are also evident. Additionally, the technology of Mixture 2 exhibits significant product-specific scale effects for commercial loans, and a comparison of these measures for all three models suggests that the effects of pooling can be quite misleading. Finally, while the scope measures for all three models are usually insignificant, the general appearance of the results again suggests that the technology inferred from the standard regression results does not resemble either of the technologies suggested by the mixture procedure.

The procedure by which the mixture estimations sorted firms into two groups is not easily duplicated by noting qualitative differences between firms. A number of variables available but not used in the final results reported in this study, including stock or mutual ownership status and state or federal charter status of sample firms, cannot even approximately duplicate the partition implied by the mixture results. When firms were sorted into one group or the other by analyzing the normalized or absolute sizes of the prediction errors implied by the mixture cost functions, no obvious distinguishing firm characteristics could be identified.

TABLE 5.—PREDICTED COSTS FOR FFC QUADRATIC COST FUNCTION MODEL AS A PERCENTAGE OF SAMPLE MEAN COSTS

Output Scaling (% of Mean)	Pooled	Mixture 1	Mixture 2
50%	0.50317	0.47429	0.69179
75%	0.74676	0.70973	0.91576
100%	0.99013	0.94508	1.14000
200%	1.9615	1.88567	2.03967
300%	2.9296	2.82492	2.94357

Note: Sample mean costs = $7,984,000. Input prices set at sample mean values. Output scaling refers to proportional scaling of the sample mean output vector.

V. Finite Mixture Techniques and Frontier Estimation

While the estimation of cost functions using traditional techniques is widespread, recent years have seen growing interest in both statistical cost and production frontier estimation and related procedures which construct graphs of technologies in a nonstochastic, linear programming setting. This literature (e.g., Forsund and Hjalmarsson (1974), Forsund and Jansen (1977), Färe, Grosskopf, and Lovell (1985), Lee and Tyler (1978), and Greene (1982)) has been at least partially motivated by the recognition that, while a "cost function" is by definition the solution to a minimization exercise, traditional cost function estimation continues to rely on stochastic specifications that do not take this into account. The interpretation of traditional cost functions estimated using typical disturbance structures is thus quite different from that attached to the results of frontier procedures.

The finite mixture technique presented in this paper is clearly an extension of traditional procedures, and we have in fact interpreted our results in this light. Some evidence on the relationship between mixture and frontier techniques, however, is offered by comparing the predicted costs obtained from a mixture and corresponding pooled estimation. Table 5 utilizes estimation results for the FFC model to calculate predicted costs, as a percentage of sample average costs, for various scalings of the sample mean output vector using sample average input prices. We find that the technology represented by Mixture 1 is lower cost than that given by both Mixture 2 and the pooled model over a wide range of output scalings. In fact, the technology implied by the Mixture 1 relationship has the appearance of a minimum cost "frontier" for a large range of outputs and input prices.[17] While this result seems sensible, the finite mixture procedure itself does not impose any specific relationship between the component cost functions, which could "cross"

[17] One can find extreme values for input prices, however, for which Mixture 2 represents a lower cost technology, though the results of table 5, combined with the scale results from table 4, lead to the speculation that Mixture 2 represents an "older" technology which is being supplanted by Mixture 1. This conjecture, however, is beyond the scope of our analysis.

one another repeatedly through the range of relevant outputs and input prices. Additionally, unlike frontier techniques, the mixture procedures used in this paper organize observations "not on the frontier" by estimating a typical cost function from which these observations were presumably generated. This fundamental methodological difference could be of some significance when one of the goals of estimation is to facilitate predictions on the effects of product line or other regulations on industry costs and performance.

VI. Conclusion

This paper presented and illustrated a technique of estimation, based on the theory of finite mixture distributions, which allows for the simultaneous existence of multiple technologies of production when the researcher does not know which observations correspond to which technologies. The technique, which is a generalization of traditional cost analysis, produces estimates of the proportions of firms using one or another technology, allows limited testing for the number of technologies concurrently in use, and facilitates comparison between technologies. The mixture methodology was illustrated by estimating both pooled and two component normal mixture specifications using the translog and flexible fixed cost quadratic cost models for a large sample of savings and loan associations operating in 1988. The statistical results strongly support the contention that this industry is characterized by multiple technologies of production. Further, the cost functions estimated by the traditional and mixture procedures exhibited quantitative and qualitative differences, suggesting that traditional cost estimation may produce misleading results in the presence of multiple technologies of production.

A number of developments and extensions of these results are possible. First, while we examined only two component mixture models, the potential number of components (and, hence, underlying technologies) is limited only by considerations of identifiability and tractability. The reader is warned, however, that greater numbers of components can present enormous practical difficulty in estimation, and that testing for the existence of mixtures with higher numbers of components is problematic. Second, while we specified identical functional forms for the cost functions used in estimation, this restriction is not necessary. Third, while we feel cost or production function estimation constitutes a natural application of the mixture methodology, the procedure is quite general and may be liable to profitable application elsewhere. Finally, while the mixture results presented here suggest a certain potential practical kinship between mixture and frontier estimation, general conclusions on this point remain an unresolved issue deserving further investigation.

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